

# Нелинейное моделирование и управление гибридными БЛА

## Nonlinear Modeling and Control of Hybrid UAVs

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## Outline of my talk...

- Introduction
- Hybrid UAV platforms
- Dynamics modeling
- Flight control system
- Some applications
- Concluding remarks



## План моего выступления

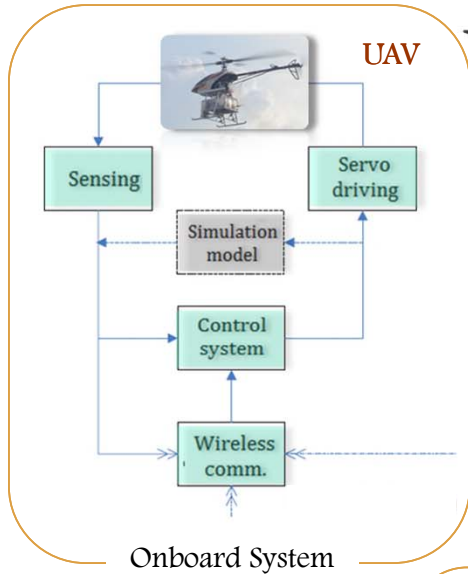
- Введение
- Гибридные платформы БПЛА
- Моделирование динамики
- Система управления полетом
- Некоторые приложения
- Заключительные замечания



# An unmanned system architecture

# Архитектура беспилотной системы

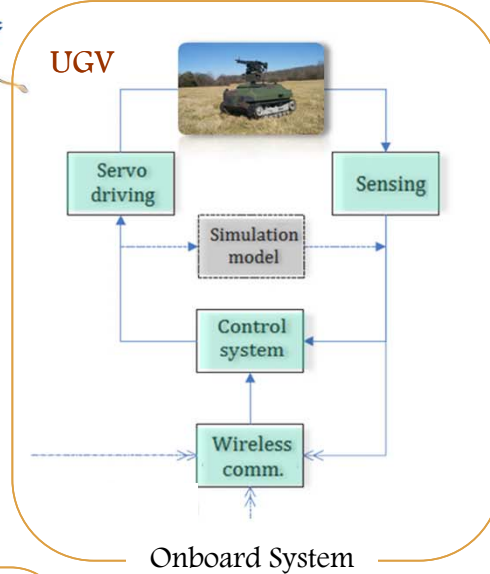
Беспилотные летательные аппараты



Data Link



UGV



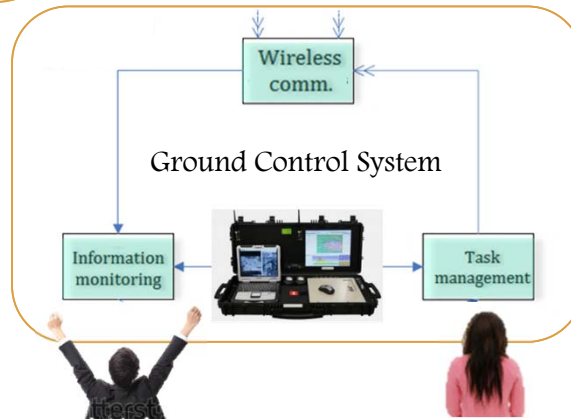
Беспилотные наземные машины

Беспилотное надводное судно

USV



Ground Control System



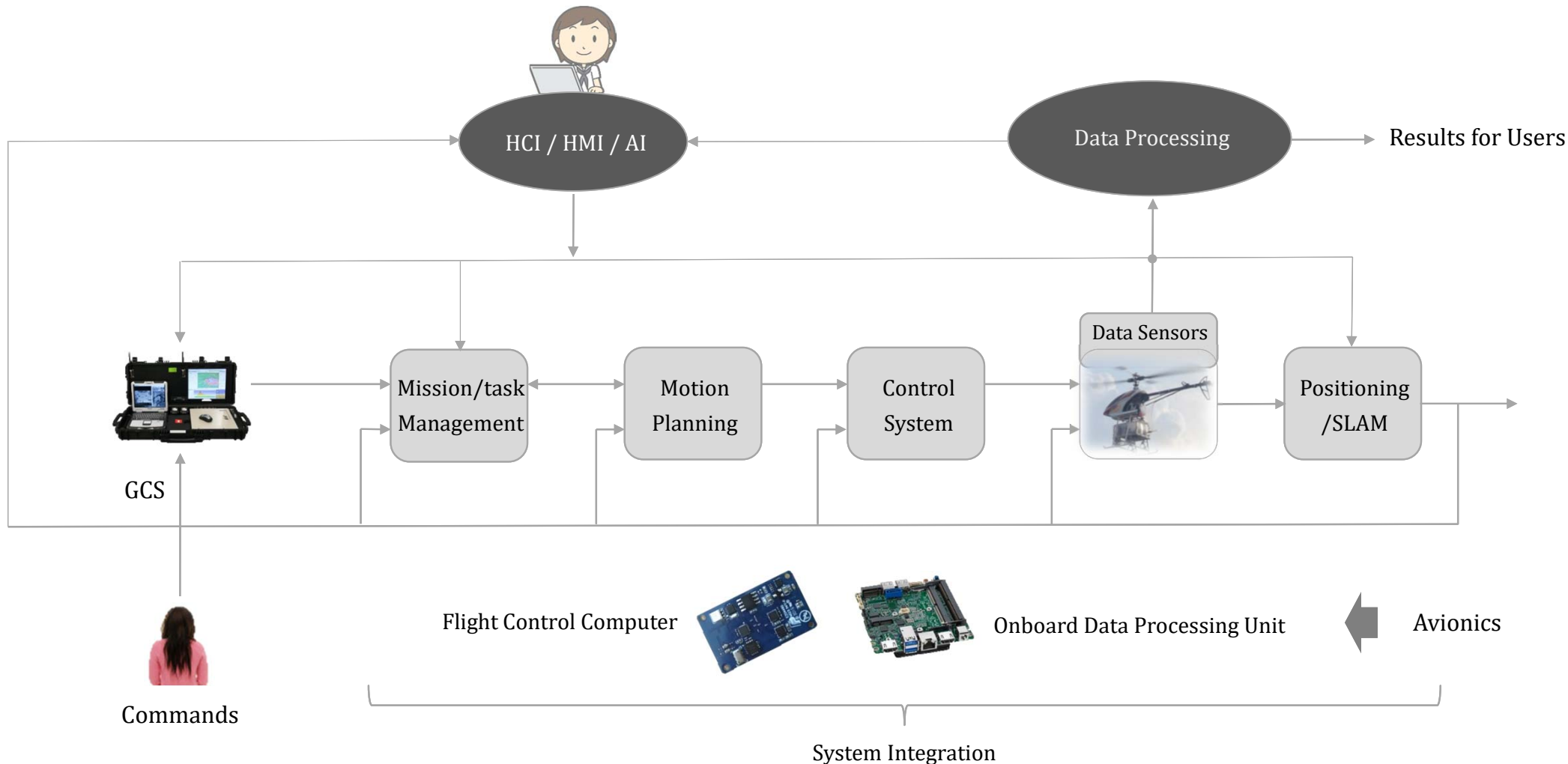
UUV



Беспилотное подводное судно

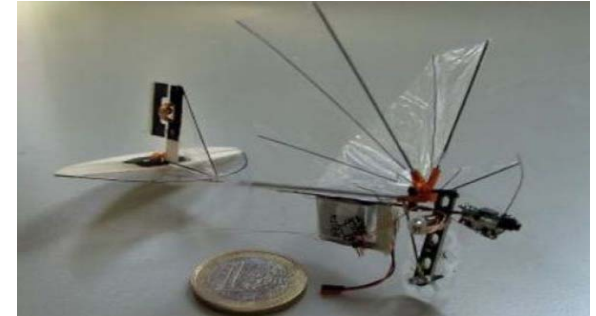
# Internal framework of an intelligent autonomous UAS...

## Внутренний каркас интеллектуального автономного БПЛА





# Some common/uncommon drones...    некоторые обычные/необычные дроны





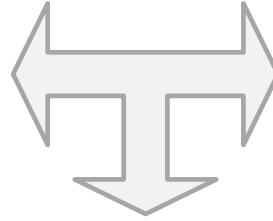
## Why hybrid UAVs?



Long Range  
Flight Efficiency



## Почему гибридные БПЛА?



An aircraft with VTOL and  
cruise flight capability



VTOL Maneuverability





## Some existing hybrid UAVs

## Некоторые существующие гибридные БЛА





# Evolution of our hybrid UAVs...

# Эволюция наших гибридных БПЛА



.....?





# Dynamics modeling...

# Моделирование динамики

➤ Kinematics:

$$\dot{\mathbf{P}}_n = \mathbf{R}_{n/b} \mathbf{V}_b,$$

$$\dot{\mathbf{R}}_{n/b} = \mathbf{W} \mathbf{R}_{n/b},$$

➤ Rigid body dynamics:

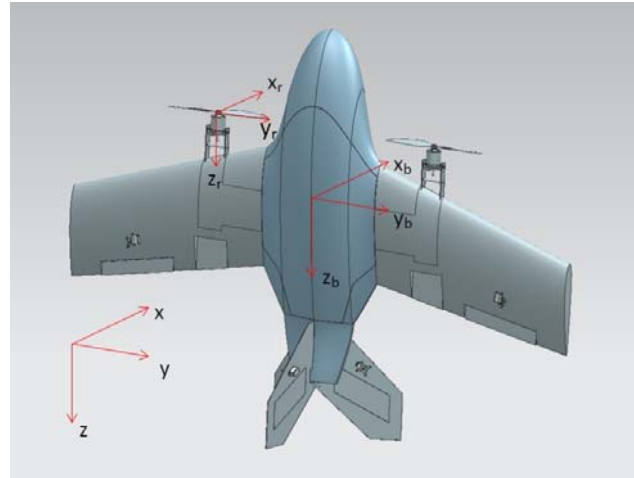
$$m \dot{\mathbf{V}}_b + \boldsymbol{\omega} \times (m \mathbf{V}_b) = \mathbf{F},$$

$$\mathbf{J} \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{J} \boldsymbol{\omega}) = \mathbf{M}$$

➤ Forces and moments:

$$\mathbf{F} = \mathbf{F}_{\text{grav}} + \mathbf{F}_{\text{prop}} + \mathbf{F}_{\text{aero}},$$

$$\mathbf{M} = \mathbf{M}_{\text{prop}} + \mathbf{M}_{\text{fin}} + \mathbf{M}_{\text{aero}},$$



**P** Position vector

**V** Velocity vector

**F** Force vector

**M** Moment vector

**R** Rotation matrix

**W** Angular velocity tensor  
**J** Moment of inertia matrix

**b** Body frame

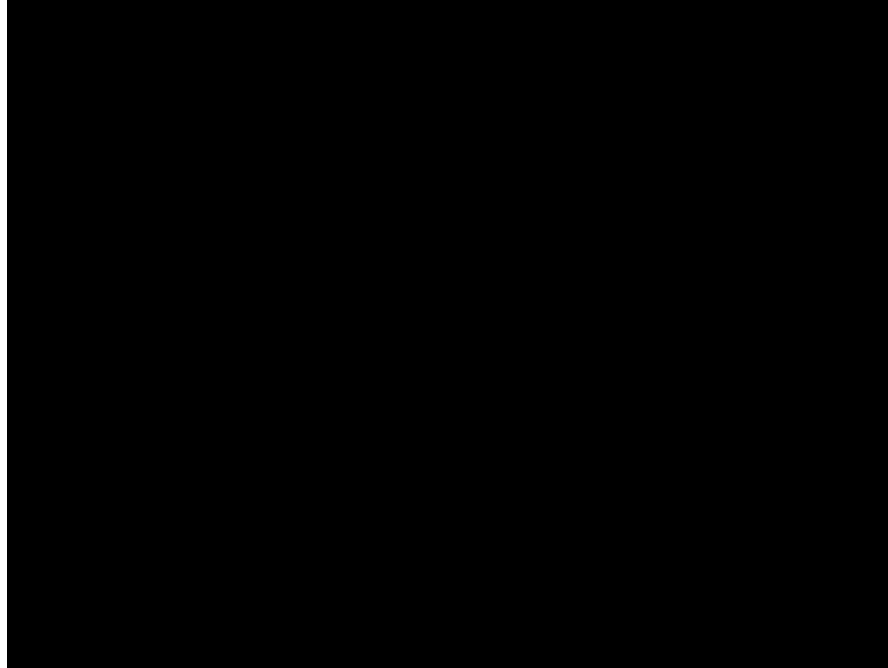
**n** Local NED frame

**m** Mass

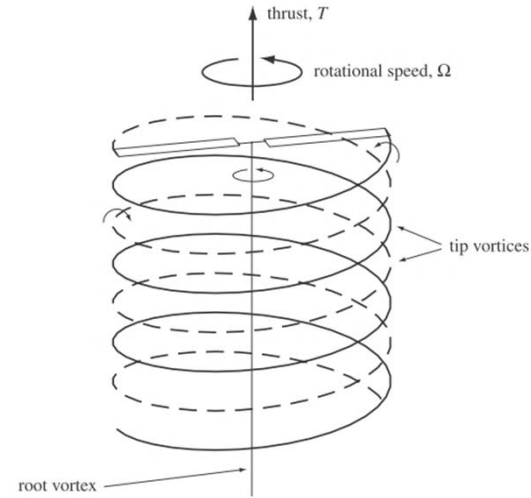
$\boldsymbol{\omega}$  Angular velocity



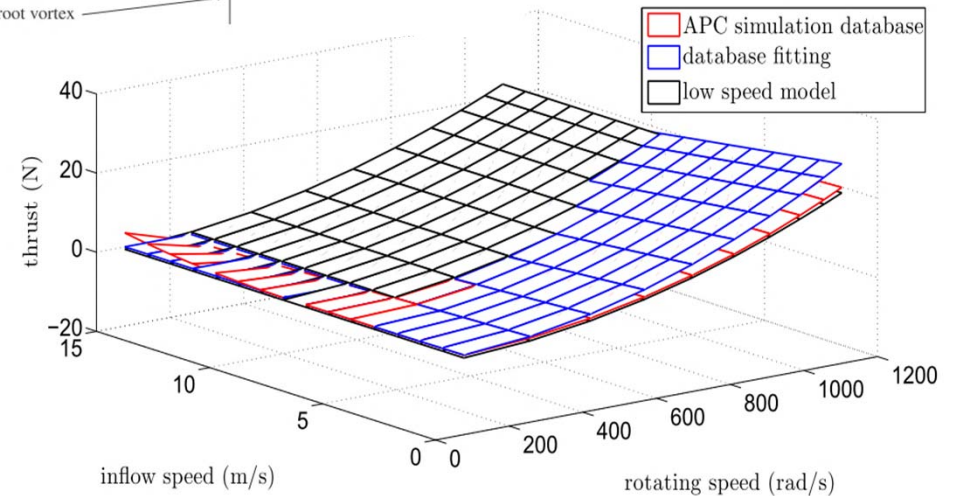
# Dynamics modeling (propeller dynamics)...



Experiment Setup



Моделирование  
динамики  
(динамика  
воздушного винта)



Simulation and  
Experimental Results



# Dynamics modeling (transition mode)...

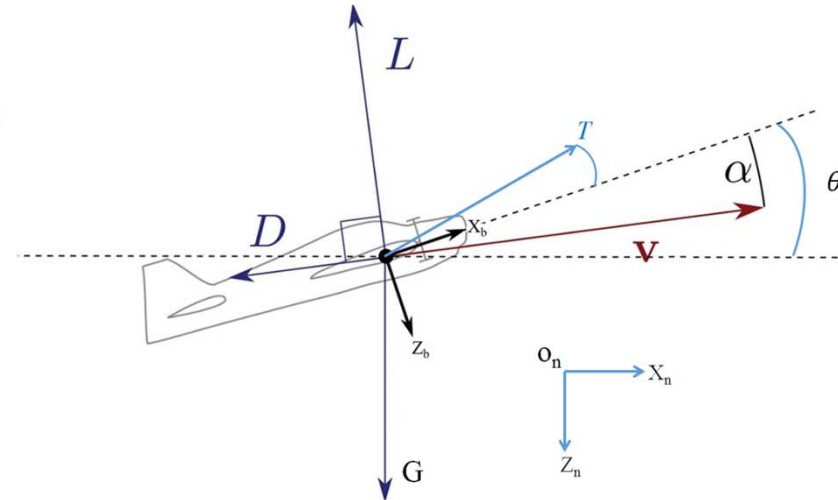
## Моделирование динамики (переходный режим)

Define state as  $\mathbf{x} = [u \ w \ q \ \theta]^T$ , input as  $\mathbf{u} = [T_u \ T_w \ T_f]^T$ , then main dynamics:

$$\begin{aligned} \dot{u} &= \frac{1}{m} F_{a,x}(\mathbf{x}) - g \sin(\theta) - qw + \frac{1}{m} T_u + \delta_u(t), \\ \dot{w} &= \frac{1}{m} F_{a,z}(\mathbf{x}) + g \cos(\theta) + qu - \frac{1}{m} T_w + \frac{1}{m} T_f + \delta_w(t), \\ \dot{q} &= \frac{1}{I_y} M_a(\mathbf{x}) + \frac{l_m}{I_y} T_w + \frac{l_f}{I_y} T_f + \delta_q(t), \\ \dot{\theta} &= q, \end{aligned}$$

where 
$$\begin{bmatrix} F_{a,x}(\mathbf{x}) \\ F_{a,z}(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ -\cos\alpha & -\sin\alpha \end{bmatrix} \begin{bmatrix} L(\alpha) \\ D(\alpha) \end{bmatrix}$$

(1)



$$L(\alpha) = \frac{1}{2}(u^2 + w^2)\rho A_w C_L(\alpha),$$

$$D(\alpha) = \frac{1}{2}(u^2 + w^2)\rho A_w C_D(\alpha).$$

$$M_a(\mathbf{x}) = \frac{1}{2}(u^2 + w^2)\rho A_w C_M(\alpha).$$

$$\alpha = \text{atan2}(u/w)$$

- $\delta_q(t)$  The unknown perturbations in  $q$  dynamics
- $\delta_u(t)$  The unknown perturbations in  $u$  dynamics
- $\delta_w(t)$  The unknown perturbations in  $w$  dynamics
- $C_D(\alpha)$  Aerodynamic drag coefficient
- $C_L(\alpha)$  Aerodynamic lift coefficient
- $C_M(\alpha)$  Aerodynamic moment coefficient

- $\rho$  Air density
- $A_w$  Surface area of the wing
- $g$  Gravity constant
- $I_y$  Moment of inertia of KH-Lion in  $Y$ -direction
- $l_f$  Distance from the tail fin center to the CG along  $X_b$
- $l_m$  Distance from motor to the CG along  $X_b$
- $m$  Mass of KH-Lion

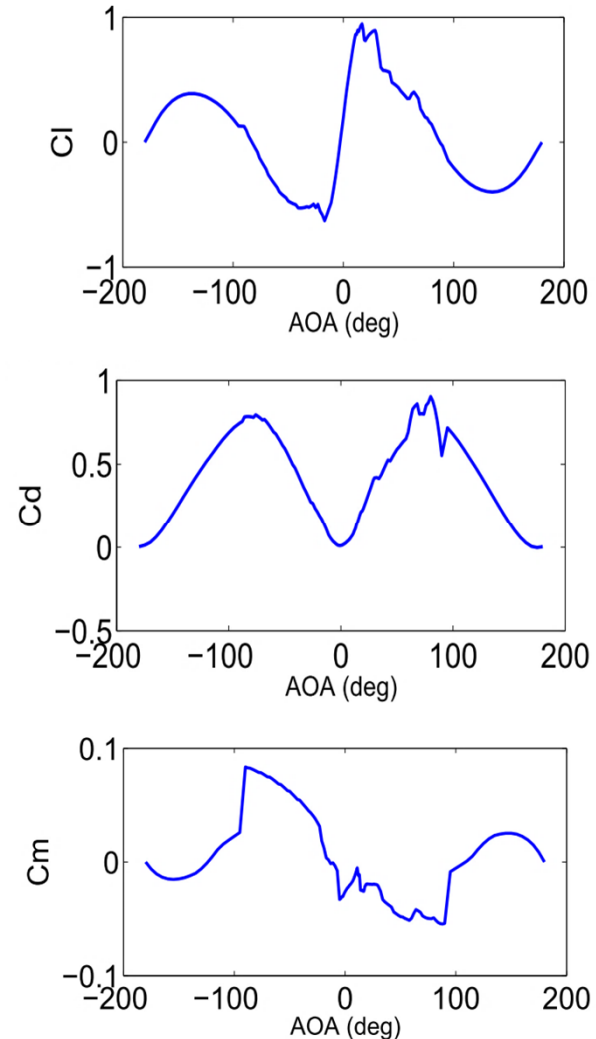
- $\gamma_v$  Vectoring thrust titling angle
- $\gamma_f$  Tail fin control surface deflection angle
- $\theta$  Pitch angle of KH-Lion
- $q$  Angular speed in pitch direction
- $T_f$  Force generated by the tail fin control surfaces in  $Z_b$  direction
- $T_u$  Thrust decomposition in  $X_b$ -axis direction
- $T_w$  Thrust decomposition in  $Z_b$ -axis direction
- $u$  Velocity in the body frame  $X_b$ -axis direction
- $w$  Velocity in the body frame  $Z_b$ -axis direction



## Dynamics modeling (aerodynamics coefficients)...

- Dynamics model is highly nonlinear and complex
- Aerodynamics coefficients depend on speed and AOA
- High AOA dynamics difficult to measure and estimate
- Uncertainties & disturbance
- Input constraints
- State (velocity, acceleration, etc.) constraints

Моделирование  
динамики  
(аэродинамические  
коэффициенты)





## Flight control systems (transition mode)...

Системы управления полетом  
(переходный режим)

- Find an optimal control law for the transition
  - Discretise the feasible state and action space
  - Find optimal action for each state in state space based on DP algorithm
  - The control law for a random state is obtained by interpolating the state in the state spaces, and sum up the corresponding actions
  
- Find the optimal trajectory for the transition
  - Apply the optimal control law obtained to the model from an initial condition
  - Recording the input and state trajectory for the transition
  
- Advantages of DP algorithm
  - The complexity of the system does not affect the algorithm complexity
  - Well handled the input and state constraints



# Flight control systems (transition mode)...

Denote the system (1) as

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$$

Discretising the system:

$$\mathbf{x}[n + 1] = f_T(\mathbf{x}[n], \mathbf{u}[n])$$

To find an optimal policy:

$$\mathbf{u}[n] = \pi^*(\mathbf{x}[n])$$

which optimizing the cost function:

$$J(\mathbf{x}_0) = h(\mathbf{x}[N]) + \sum_{n=0}^{N-1} g(\mathbf{x}, \mathbf{u}, n)$$

The cost function is selected to be:

$$\begin{aligned} (\mathbf{x}[n]) &= (\mathbf{x}[n] - \mathbf{x}_c)^T Q_f (\mathbf{x}[n] - \mathbf{x}_c) \\ (\mathbf{x}, \mathbf{u}, n) &= \mathbf{u}[n]^T R_f \mathbf{u}[n], \end{aligned}$$

$\mathbf{x}_c$  Nominal state of cruise flight  
 $Q_f, R_f$  Weight matrix

Rewrite the cost function in recursive format:

$$\begin{aligned} J^*(\mathbf{x}, n) &= \min_{\mathbf{u}} [g(\mathbf{x}, \mathbf{u}, n) + J^*(\mathbf{x}[n + 1], n + 1)], \\ \pi^*(\mathbf{x}, n) &= \operatorname{argmin}_{\mathbf{u}} [g(\mathbf{x}, \mathbf{u}, n) + J^*(\mathbf{x}[n + 1], n + 1)] \end{aligned}$$

The optimal policy can be found by DP algorithm:

**Data:** State set  $S$ , action set  $A$

**Result:** For each  $s \in S$ , the optimal policy  $\pi^*(s)$  and optimal cost  $J^*(s)$

**for each state  $s \in S$  do**

$J^*(s) \leftarrow h(s)$

**end**

**while  $J^*(s)$  not converged do**

**for each state  $s$  do**

**for each action  $a$  do**

$s' \leftarrow f(s, a);$

            do volumetric interpolation  $s'$  in  $S$  so that

$s' = \sum_{m=1}^{16} w_m s_m, s_m \in S$

$J = g(s, a) + \sum_{m=1}^{16} w_m J^*(s_m)$

**if  $J < J^*(s)$  then**

$J^*(s) \leftarrow J$

$\pi^* \leftarrow a$

**end**

**end**

**end**

**end**

Системы  
управления  
полетом  
(переходный  
режим)

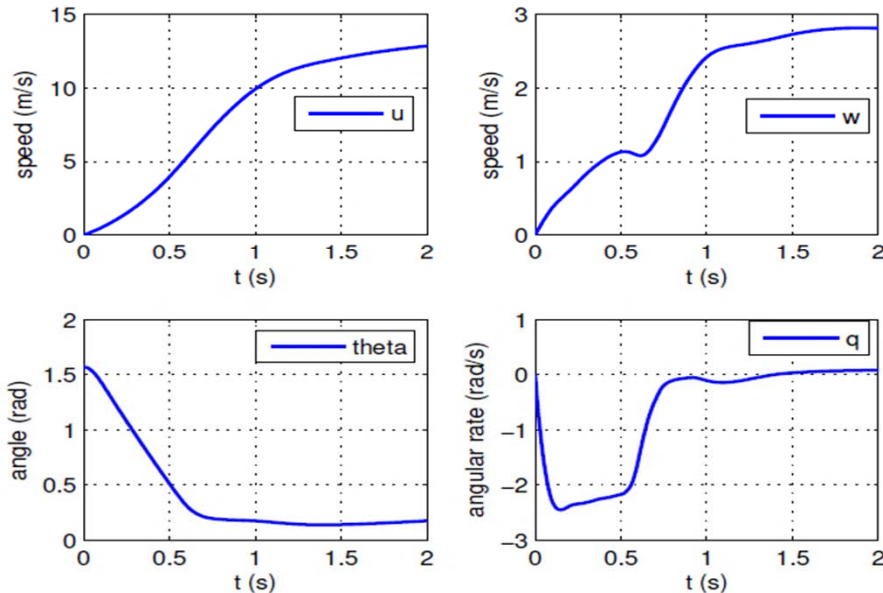


# Flight control systems (transition mode)...

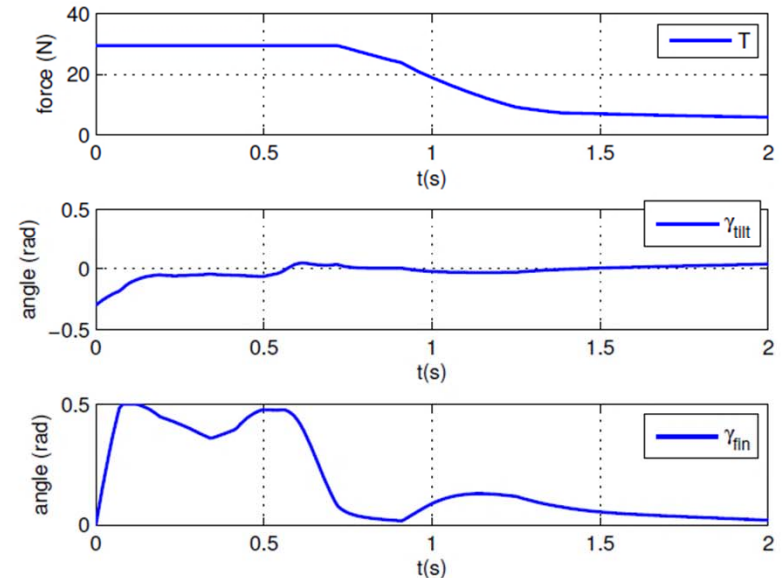
Системы  
управления  
полетом  
(переходный  
режим)

Once the  $\pi^*(s)$  is obtained, the optimal control law for state  $x$  is obtained through:

1. Do volumetric interpolation so that:  $x = \sum_{m=1}^{16} w_m s_m, s_m \in S$ .
2. Obtain the optimal control input:  $u = \sum_{m=1}^{16} w_m \pi^*(s_m)$ .



State trajectories for forward transition



Input trajectories for forward transition



## Flight control systems (transition mode)...

Системы управления полетом  
(переходный режим)

Once we obtain the optimal trajectory:

$$v^*(t) = (u^*(t), w^*(t), q^*(t), \theta^*(t), T_u^*(t), T_w^*(t), T_f^*(t))$$

The error dynamics for tracking the trajectory is:

$$\begin{aligned}\dot{\tilde{u}} &= \Psi_u(\tilde{u}, \tilde{w}, \tilde{q}, \tilde{\theta}, t) + \frac{1}{m}\tilde{T}_u, \\ \dot{\tilde{w}} &= \Psi_w(\tilde{u}, \tilde{w}, \tilde{q}, \tilde{\theta}, t) - \frac{1}{m}\tilde{T}_w + \frac{1}{m}\tilde{T}_f, \\ \dot{\tilde{q}} &= \Psi_q(\tilde{u}, \tilde{w}, \tilde{q}, \tilde{\theta}, t) + \frac{l_m}{I_y}\tilde{T}_w + \frac{l_t}{I_y}\tilde{T}_f, \\ \dot{\tilde{\theta}} &= \tilde{q},\end{aligned}$$

Treat the error terms  $\Psi(u)$ ,  $\Psi(w)$ ,  $\Psi(q)$  as lumped disturbances, then the nominal dynamics becomes is an LTI system and any linear control law can be designed to regulate the errors to zero.





# Fully autonomous flight test...

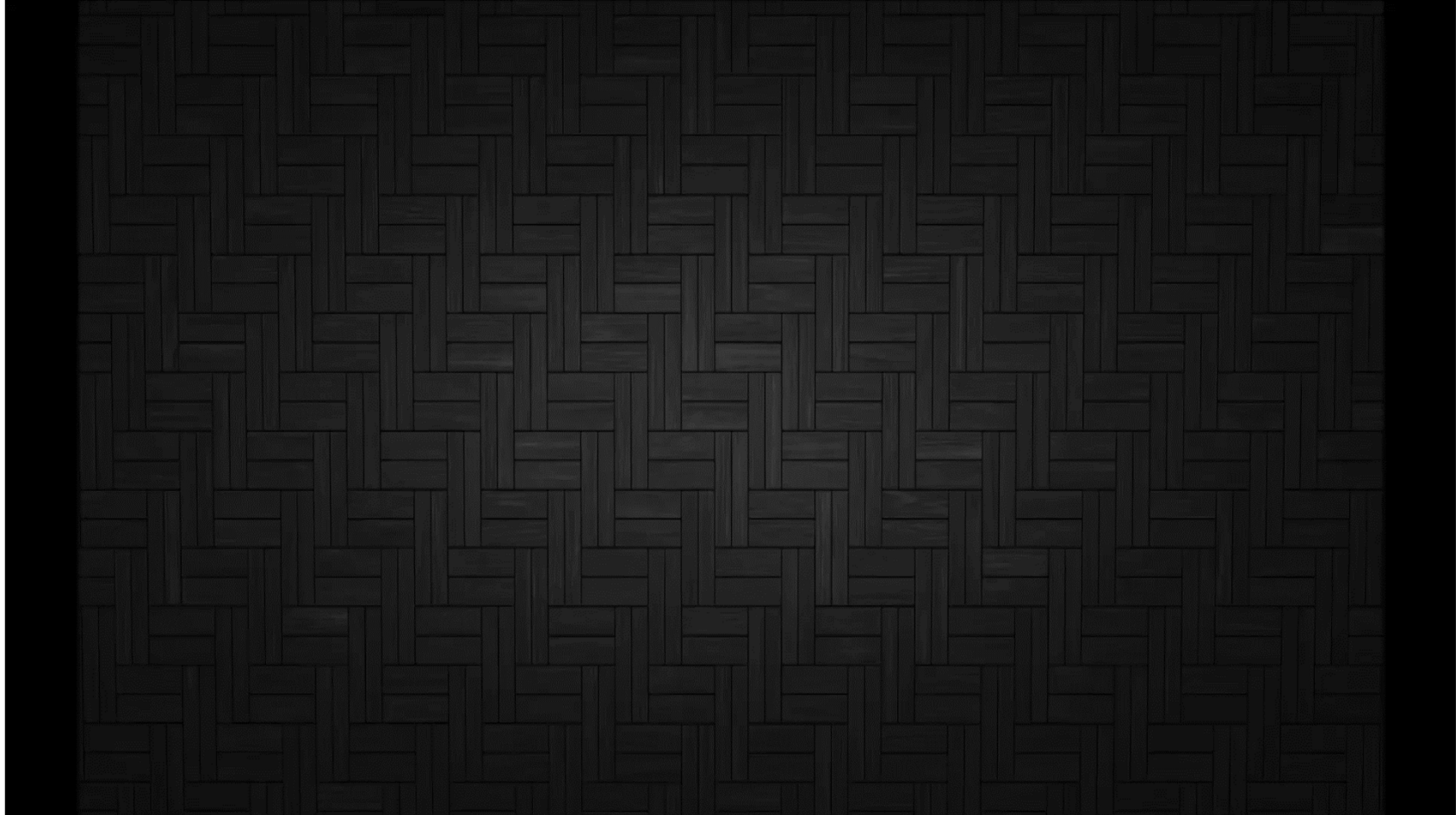
# Полностью автономные летные испытания





Auto-landing on moving platform...

Автоматическая посадка на движущуюся платформу





## Concluding remarks...

## Заключительные замечания

We invite nonlinear systems and control experts to help tackling our problem...



**ПОМОГИТЕ!**

Мы приглашаем специалистов по нелинейным системам и управлению для решения нашей проблемы.



Огромное спасибо!  
Thank You!



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